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Optimal Deterioration Control in Perishable Inventory Systems with Trade Credit and Demand Dependent on Time, Price, Advertisement, and Quality

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Abstract: This study presents a Deterioration Control Decision Support System (DCDSS) tailored for retailers dealing with branded organic fruits or vegetables, where product perishability significantly impacts profitability. The model considers a trade credit environment in which the supplier allows a delayed payment period, and shortages are permitted with partial backlogging. Demand is modelled as a function of time, selling price, advertisement effort, and the product's deterioration rate capturing real-world dynamics of consumer behavior for highly perishable, quality-sensitive goods. The proposed system integrates the effects of freshness preservation technology, where the deterioration rate is controllable through investments that influence holding costs. A quality indicator is employed to quantify the impact of preservation on deterioration, linking technical effort with marginal cost. The decision model aims to minimize total inventory cost while optimizing the deterioration rate and managing trade-offs among marketing efforts, pricing strategies, and credit policies. Numerical experiments demonstrate the sensitivity of optimal solutions to key parameters, offering actionable insights for retailers seeking to maximize profit and maintain product quality in competitive perishable goods markets.

keywords: Trade credit, partial backlogging, deterioration dependent demand

1. Introduction and Literature Review

The retailing of perishable goods, particularly branded organic fruits and vegetables, presents complex challenges due to their inherent deterioration over time and sensitivity to external factors such as price, advertisement, and storage conditions. Retailers must make informed decisions to balance freshness, availability, cost, and consumer demand in a dynamic marketplace. Moreover, the inclusion of trade credit arrangements where suppliers allow deferred payment and the possibility of shortages adds further complexity to inventory management. In traditional inventory models,

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deterioration is often treated as a fixed parameter; however, in practice, the deterioration rate can be influenced through preservation technologies, such as cold storage, modified atmosphere packaging, or humidity control systems. These technologies come at a cost, creating a trade-off between investment in quality maintenance and overall inventory cost. This necessitates a decision-making framework that allows for controllable deterioration within the broader scope of demand responsiveness and financial considerations. Demand for branded organic produce is not only time-sensitive but also highly dependent on perceived quality, price competitiveness, and marketing effectiveness. Consumers tend to favor fresher products and are highly responsive to visible stock, promotional efforts, and price changes. Therefore, modelling demand as a function of time, selling price, advertisement effort, and deterioration rate provides a realistic and holistic representation of customer behavior in modern perishable retail environments. This paper develops a Deterioration Control Decision Support System (DCDSS) that assists retailers in optimizing inventory decisions under a trade credit policy and partial backlogging during shortages. The model integrates a freshness quality indicator that connects preservation efforts to marginal holding costs and deterioration rates. By minimizing the total inventory cost while accounting for dynamic demand and financial variables, the system offers practical insights for enhancing profitability and customer satisfaction in quality-sensitive markets. The management of perishable inventory has been extensively studied in operations research, particularly focusing on deterioration, demand behavior, and replenishment strategies. Early models often assumed a constant deterioration rate (Ghare & Schrader, 1963), but such assumptions are increasingly unrealistic in modern supply chains, where technology can influence preservation. More recent studies introduce controllable deterioration, where investment in preservation methods can slow down spoilage (Hsieh et al., 2008; Sarkar, 2012), offering flexibility in inventory planning for perishable goods. The concept of trade credit, introduced into inventory systems by Goyal (1985), allows retailers to delay payments to suppliers, which can significantly influence order quantities and profitability. Since then, researchers have extended these models to consider various credit structures, shortages, and deterioration (Teng et al., 2009; Chung & Huang, 2003). Trade credit policies are particularly useful in high-risk perishable markets where immediate liquidity can be a constraint. Demand modelling in perishable inventory systems has evolved from static to dynamic forms, incorporating time dependency (Wee, 1995), price sensitivity (Bhunia & Shaikh, 2011), and more recently, *advertisement and quality sensitivity* (Mondal et al., 2017). In highly competitive sectors like branded organic produce, advertisement significantly influences consumer buying behavior. Simultaneously, quality perception—linked to deterioration rate plays a vital role in shaping demand (Zhou et al., 2012). Quality-dependent demand models, where consumers favor fresher items even at equal prices, align closely with real-world retail dynamics. Several works (e.g., Sana, 2008; Sarkar et al., 2015) have analyzed inventory models with shortages and partial backlogging, where some customers are willing to wait for restocking while others turn away. Backlogging models are essential in perishable systems where stockouts are not uncommon due to short product lifecycles and unpredictable demand. Tayal et al. (2016) developed an inventory model in which they consider perishable products and also show effect of preservation technology and trade credit. Singh et al. (2021) established an inventory model in which they consider supply chain with the effect of carbon emission and trade credit policy. Singh, *et al*. (2022) formulate inventory model for price-sensitive demand and preservation investment. They also, considered the effect of partial backlogging with low carbon. Padiyar *et al* (2022) formulate inventory model for deteriorating items with the effect of inflation. Singh, *et al.* (2022) formulate the inventory model with impact of preservation technology investment. They also consider different carbon emission policies. Padiyar *et al* (2022) formulates three echelon supply chain with the effect of inflation. Singh, & Singh (2023) formulate inventory model with the impact of circular economy on sustainable inventory model. They integrate renewable energy and green environment. Goel *et al* (2024) developed inventory model with partial trade credit. Patra *et al.* (2024) created an inventory model in which they consider credit policy with preservation technology but do not consider carbon emission effect. While existing literature has explored deterioration control, trade credit, and quality- or price-sensitive demand individually, there is limited work that integrates all these factors simultaneously, especially with advertisement-dependent and time-varying demand. Furthermore, few models provide a practical decision-support framework that helps retailers balance deterioration investment, demand stimulation, and trade credit financing under shortage conditions.

This study builds on these research gaps by proposing a comprehensive decision support system for perishable goods retailing. It contributes to the literature by modelling demand as a function of time, selling price, advertisement effort, and deterioration rate, while also integrating trade credit financing and controllable deterioration in the presence of shortages.

The remainder of this paper is organized as follows: Section 2 presents the assumptions and section 3 represents notations. Section 4 presents mathematical model. Section 5 discusses solution methodology and optimization. Section 6 provides numerical analysis and graphical representation. Section 7 discusses sensitivity experiments. Section 8 presents observations. Finally, Section 9 concludes with managerial implications and future research directions.

2. Notations and Assumptions

The following assumptions are considered for this model-

1. The demand for this model is considered as

   $D(p,t,\theta) = (a - bp + ct)A^{\gamma} - d\theta$

2. The study is restricted to a single-product inventory system.
3. The indicator of fresh quality technology is $\mu = \frac{C_h'}{C_h''}$. Jani et al. (2023)

   Where C_h'= controllable marginal cost of holding with preservation

   C_h'' = the maximum cost of holding including preservation which denotes the highest-level effort of C_h'.

4. The supplier offers a trade credit period of M years to the retailer. During the interval $[0, M]$, the retailer earns interest I_e on the revenue from sold items. However, for the remaining period $[M, T]$, interest I_c is charged on the value of unsold inventory.
5. The proportion of partially backlogged shortages is denoted by $B(t)$ a differentiable and decreasing function of time t. Where $B(t) = e^{-\delta(T-t)}$, δ is backlogging parameter and $(T - t)$ is the waiting time for the next replacement.
6. Inflation is also considered.
7. Carbon emission is also considered for holding inventory and deteriorated inventory

2.2 Notations

The following notations are used for this model

Table 1 Notations and their descriptions

Parameters	Units	Descriptions
a	-	Scaling parameter
b	-	Scaling parameter
c	-	Scaling parameter
A	-	Advertisement frequency
γ		Rate of change of advertisement frequency
d		Scaling parameter
B		Backlogging parameter
Q		Total quantity
o		Ordering cost
C_h		Holding cost
C_d		Deterioration cost
S_c		Shortage cost
S_l		Lost sale cost
I_e		Interest earn
I_c		Interest charge
C_p		Quota price of carbon
z		Emission quota of carbon emission per unit time (kg)
M		Trade credit period
T_1		Time when production stop
T		Total cycle length
$I_1(t)$		Inventory level between $0 \leq t \leq T_1$
$I_2(t)$		Inventory level between $T_1 \leq t \leq T$
Decision variable		
T_1		Time when production stop
T		Total cycle length

Based on these assumptions developed an inventory model which is shown below-

3. Mathematical Formulation

This section presents the mathematical model for managing perishable products, incorporating exponentially partially backlogged shortages and demand that is dependent on the deterioration rate. Within each replenishment cycle, the inventory level declines due to both product deterioration and customer demand.

Furthermore, the inventory level over the cycle time T is illustrated in Figure 1, which visually demonstrates the combined impact of deterioration and demand on stock depletion. Due to the demand being dependent on the deterioration rate, the inventory level gradually decreases over the time interval $[0, t_1]$, eventually depleting completely at time t_1. Therefore, the inventory level with in the interval $[0, t_1]$ can be expressed as follows:

$$\frac{dI_1(t)}{dt} = -\theta I_1(t) - D(\theta, p, t), 0 \le t \le T_1 \quad (1)$$

$$\frac{dI_2(t)}{dt} = D(\theta, p, t)e^{-f(T-t)}, T_1 \le t \le T \quad (2)$$

With boundary conditions $I_1(T_1) = 0, I_2(0) = Q - B, I_2(T_1) = 0, I_2(T) = B$

Solution of these equations-

$$I_1(t) = \left((a - bp)A^\gamma - d\theta\right)\frac{e^{\theta(T_1-t)}-1}{\theta} + cA^\gamma\left(\frac{T_1e^{\theta(T_1-t)}}{\theta} - \frac{e^{\theta(T_1-t)}}{\theta^2} - \frac{t}{\theta} + \frac{1}{\theta^2}\right) \quad (3)$$

$$I_2(t) = \left((a - bp)A^\gamma - d\theta\right)\frac{e^{-f(T-t)}-e^{-f(T-T_1)}}{f} + cA^\gamma\left(\frac{te^{-f(T-t)}-T_1e^{-f(T-T_1)}}{f} - \frac{e^{-f(T-t)}+e^{-f(T-T_1)}}{f^2}\right) \quad (4)$$

And also, $B = \left((a - bp)A^\gamma - d\theta\right)\frac{1-e^{f(T-T_1)}}{f} + cA^\gamma\left(\frac{1-T_1e^{-f(T-T_1)}}{f} - \frac{1+e^{-f(T-T_1)}}{f^2}\right)$ (5)

$$Q = \left((a - bp)A^\gamma - d\theta\right)\left(\frac{e^{\theta T_1}-1}{\theta} + \frac{1-e^{f(T-T_1)}}{f}\right) + cA^\gamma\left(\frac{T_1e^{\theta T_1}}{\theta} - \frac{e^{\theta T_1}}{\theta^2} + \frac{1}{\theta^2} + \frac{1-T_1e^{-f(T-T_1)}}{f} - \frac{1+e^{-f(T-T_1)}}{f^2}\right) \quad (6)$$

Now find all inventory cost-

Ordering cost- $OC = o$ (7)

Holding cost-$HC = (C_h + C_h''(1-\theta)^\alpha)\int_0^T I_1(t)e^{-rt}dt$

$$= (C_h + C_h''(1-\theta)^\alpha)\left[\frac{(a-bp)A^\gamma-d\theta}{\theta}\left(\frac{e^{-rT_1}-e^{\theta T_1}}{-\theta-r} + \frac{e^{-rT_1}-1}{r}\right) + cA^\gamma\left(\frac{T_1(e^{-rT_1}-e^{\theta T_1}}{-\theta(\theta+r)} + \frac{e^{\theta(T-T_1)-rT_1}-e^{\theta T_1}}{\theta^2(\theta+r)} + \frac{1}{\theta}\left(\frac{T_1e^{-rT_1}}{r} + \frac{e^{-rT_1}-1}{r^2}\right) - \frac{e^{-rT_1}-1}{r\theta^2}\right)\right] \quad (8)$$

Deteriorating cost- $HC = C_d\left(Q - \int_0^{T_1} I_1(t)e^{-rt}dt\right)$

$$= C_d\left(Q - \left(\left((a-bp)A^{\gamma} - d\theta\right)\frac{1-e^{-rT_1}}{r} + cA^{\gamma}\left(\frac{T_1e^{-rT_1}}{r} + \frac{e^{-rT_1}-1}{r^2}\right)\right)\right) \quad (9)$$

Shortage cost- $SC = -S_c\int_{T_1}^{T} I_2(t)e^{-rt}\,dt$

$$= -S_C\left[\left((a-bp)A^{\gamma} - d\theta\right)\left(\frac{1}{f}\left(\frac{e^{-rT} - e^{-f(T-T_1)-rT_1}}{f-r} + \frac{e^{-f(T-T_1)-rT} - e^{-rT_1}}{r}\right)\right) + CA^{\gamma}\left(\frac{1}{f}\left(\frac{Te^{-rT} - T_1e^{-f(T-T_1)-rT_1}}{f-r} - \frac{e^{-rT} - e^{-f(T-T_1)-rT_1}}{(f-r)^2} + \frac{T_1e^{-f(T-T_1)-rT} - T_1e^{-f(T-T_1)-rT_1}}{r}\right) - \frac{1}{f^2}\left(\frac{e^{-rT} - e^{-f(T-T_1)-rT_1}}{f-r} + \frac{e^{-f(T-T_1)-rT} - e^{-f(T-T_1)-rT_1}}{r}\right)\right)\right] \quad (10)$$

Lost sale cost- $LSC = C_l\int_{T_1}^{T} D(p,t,\theta)\left(1-e^{-\xi(T-t)}\right)e^{-rt}dt$

$$= C_l\left[\left((a-bp)A^{\gamma} - d\theta\right)\left(\frac{e^{-rT}-e^{-rT_1}}{-r} - \frac{1-e^{\xi(T-T_1)}}{\xi}\right) + cA^{\gamma}\left(\frac{Te^{-rT}-T_1e^{-rT_1}}{-r} - \frac{e^{-rT}-e^{-rT_1}}{r^2} - \frac{T-T_1e^{-\xi(T-T_1)}}{\xi} + \frac{1-e^{-\xi(T-T_1)}}{\xi^2}\right)\right] \quad (11)$$

Now calculate carbon emission-

Total carbon emission due to holding inventory-

$$CE_h = C_{eh}\left[\frac{(a-bp)A^{\gamma}-d\theta}{\theta}\left(\frac{e^{-rT_1}-e^{\theta T_1}}{-\theta-r} + \frac{e^{-rT_1}-1}{r}\right) + cA^{\gamma}\left(\frac{T_1(e^{-rT_1}-e^{\theta T_1}}{-\theta(\theta+r)} + \frac{e^{\theta(T-T_1)-rT_1}-e^{\theta T_1}}{\theta^2(\theta+r)} + \frac{1}{\theta}\left(\frac{T_1e^{-rT_1}}{r} + \frac{e^{-rT_1}-1}{r^2}\right) - \frac{e^{-rT_1}-1}{r\theta^2}\right)\right] \quad (12)$$

Total carbon emission due to deteriorating inventory-

$$CE_d = C_{ed}\left(Q - \left(\left((a-bp)A^{\gamma} - d\theta\right)\frac{1-e^{-rT_1}}{r} + cA^{\gamma}\left(\frac{T_1e^{-rT_1}}{r} + \frac{e^{-rT_1}-1}{r^2}\right)\right)\right) \quad (13)$$

Total carbon emission is $CE = CE_h + CE_d$

$$CE = C_{eh}\left[\frac{(a-bp)A^{\gamma}-d\theta}{\theta}\left(\frac{e^{-rT_1}-e^{\theta T_1}}{-\theta-r}+\frac{e^{-rT_1}-1}{r}\right)+cA^{\gamma}\left(\frac{T_1(e^{-rT_1}-e^{\theta T_1}}{-\theta(\theta+r)}+\frac{e^{\theta(T-T_1)-rT_1}-e^{\theta T_1}}{\theta^2(\theta+r)}+\frac{1}{\theta}\left(\frac{T_1e^{-rT_1}}{r}+\frac{e^{-rT_1}-1}{r^2}\right)-\frac{e^{-rT_1}-1}{r\theta^2}\right)\right]+C_{ed}\left(Q-\left(\left((a-bp)A^{\gamma}-d\theta\right)\frac{1-e^{-rT_1}}{r}+cA^{\gamma}\left(\frac{T_1e^{-rT_1}}{r}+\frac{e^{-rT_1}-1}{r^2}\right)\right)\right) \quad (14)$$

Now the carbon emission cost under cap-and-trade policy is-

$$Cap^c = C_p(CE-z)$$

$$= C_p\left(C_{eh}\left[\frac{(a-bp)A^{\gamma}-d\theta}{\theta}\left(\frac{e^{-rT_1}-e^{\theta T_1}}{-\theta-r}+\frac{e^{-rT_1}-1}{r}\right)+cA^{\gamma}\left(\frac{T_1(e^{-rT_1}-e^{\theta T_1}}{-\theta(\theta+r)}+\frac{e^{\theta(T-T_1)-rT_1}-e^{\theta T_1}}{\theta^2(\theta+r)}+\frac{1}{\theta}\left(\frac{T_1e^{-rT_1}}{r}+\frac{e^{-rT_1}-1}{r^2}\right)-\frac{e^{-rT_1}-1}{r\theta^2}\right)\right]+C_{ed}\left(Q-\left(\left((a-bp)A^{\gamma}-d\theta\right)\frac{1-e^{-rT_1}}{r}+cA^{\gamma}\left(\frac{T_1e^{-rT_1}}{r}+\frac{e^{-rT_1}-1}{r^2}\right)\right)\right)-z\right) \quad (15)$$

In this model, a supplier provides goods to a retailer under a trade credit policy, which allows for a delayed payment period. The analysis includes the interest earned by the retailer, the interest charged by the supplier, and the overall total cost function. Two cases arise-

Case 1- $0 \le M \le T_1$

Case 2- $0 \le T_1 \le M$

For case 1 calculate interest earn and interest charge-

In this scenario, the credit phase occurs before inventory is depleted in the system. This implies that a supplier provides a period of credit M to a retailer before a time T_1. During the interval $[0, M]$, the store earns interest on the profit obtained from sustaining shortages in the previous cycle (Figure).

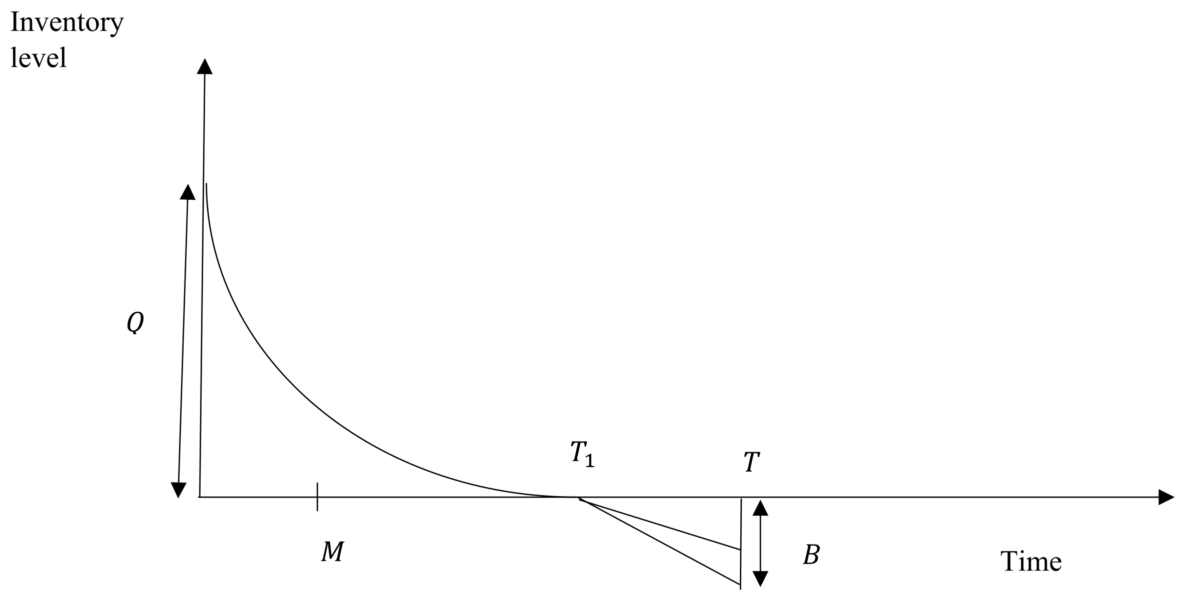

Fig 1- inventory level with respect to time for case-1

Interest earn $IE = I_e p \int_0^M D(p,t,\theta) t e^{-rt} dt + I_e p \int_0^M B e^{-rt} dt$

$$= I_e p \left[\left((a - bp)A^\gamma - d\theta \right) \left(\frac{Me^{-rM}}{-r} - \frac{e^{-rM} - 1}{r^2} \right) + cA^\gamma \left(\frac{M^2 e^{-rM}}{-r} - \frac{2M\, e^{-rM}}{r^2} + \frac{2(e^{-rM} - 1)}{r^3} \right) \right]$$

$$+ I_e p \left[\frac{e^{-rM} - 1}{r} \right] \tag{16}$$

The store gains interest throughout the interval $[0, M]$. However, after the credit time M, they charge for the interest on unsold products in the time period $M, T_1]$. Retailers' interest charges can be computed as follows:

$$IC = cI_c \int_M^{T_1} I_1(t) e^{-rt} dt$$

$$= cI_c\left[\left((a-bp)A^{\gamma}-d\theta\right)\left(\frac{1}{\theta}\left(\frac{e^{\theta(T_1-M)}-1}{\theta}-T_1+M\right)\right)+cA^{\gamma}\left(\frac{T_1(e^{\theta(T_1-M)}-1)}{\theta^2}+\frac{1-e^{\theta(T_1-M)}}{\theta^3}-\frac{T_1^2-M^2}{2\theta}+\frac{T_1-M}{\theta^2}\right)\right] \quad (17)$$

For case 2 calculate interest earn and interest charge-

In this scenario, the credit period begins once the inventory is removed from the system. This indicates that a supplier grants a credit term M to a retailer after $t = T_1$. The shop sold all of the items supplied from the supplier on credit for the time period $[0, M]$. The interest charged to a retailer was zero (i.e., $IC = 0$) under the system.
The store only gets interest on sold items during the time period $[0, M]$ and on increased inventory during the interval $[T_1, M]$ (refer to Figure.2)

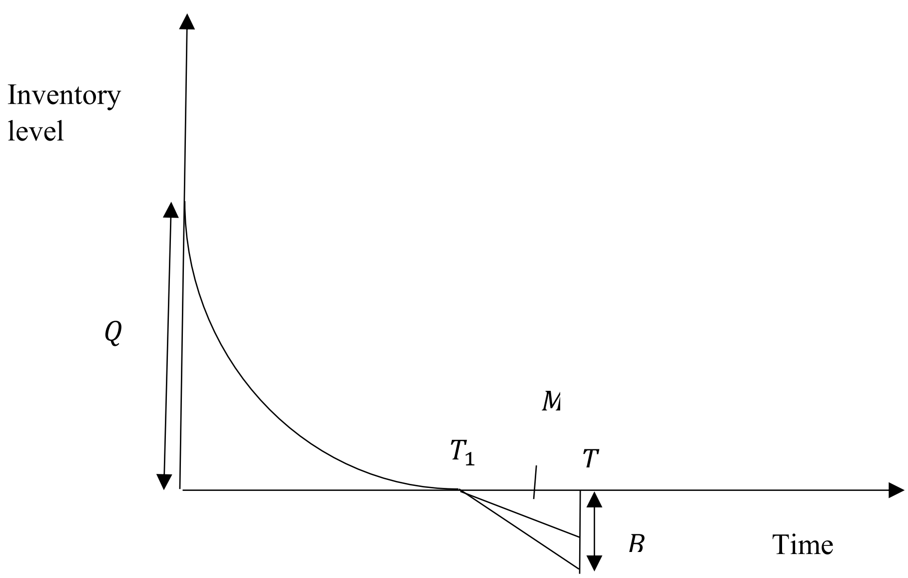

Fig 2- inventory level with respect to time for case-2

$$IE = pI_e \int_0^{T_1} tD(p,t,\theta)e^{-rt}dt + pI_e \int_0^{M} Be^{-rt}dt \ + pI_e \int_0^{T_1} D(p,t,\theta)(M-T_1)e^{-rt}dt$$

$$= pI_e\left(\left((a-bp)A^{\gamma}-d\theta\right)\left(\frac{T_1e^{-rT_1}}{-r}-\frac{e^{-rT_1}-1}{r^2}\right)+cA^{\gamma}\left(\frac{T_1e^{-rT_1}}{-r}-\frac{2T_1e^{-rT_1}}{r^2}-\frac{2(e_1^{-rT}-1)}{r^3}\right)\right)+$$

$$pI_e\left(\frac{e^{-rM}-1}{-r}\right)+pI_e\left(\left((a-bp)A^{\gamma}-d\theta\right)\left(\frac{1-e^{-rT_1}}{r}\right)+cA^{\gamma}\left(\frac{T_1e^{-rT_1}}{-r}-\frac{e^{-rT_1}-1}{r^2}\right)\right) \quad (18)$$

Now, the total cost for both case is-

$$TC = \begin{cases} TC_1 \ when\ 0 \le M \le T_1 \\ TC_2 \ when\ 0 \le T_1 \le M \end{cases}$$

Where $TC_1 = \frac{1}{T}[OC + HC + DC + SC + LSC + CE + IC - IE]$ When, $0 \le M \le T_1$

$TC_2 = \frac{1}{T}[OC + HC + DC + SC + LSC + CE + IC - IE]$ When, $0 \le T_1 \le M$

4. Solution Methodology

The following methodology is used for to show that developed model is convex.

First of all, find first order derivative with respect to decision variable such that

$\frac{dTC}{dT}$ and $\frac{dTC}{dT_1}$

And put equal to zero $\frac{dTC}{dT} = 0$ and $\frac{dTC}{dT_1} = 0$

and get the value of T and T_1.

And check principal minor of Hassian matrix.

$$H = \begin{bmatrix} \frac{\partial^2 TC}{\partial T^2} & \frac{\partial^2 TC}{\partial T_1 \partial T} \\ \frac{\partial^2 TC}{\partial T \partial T_1} & \frac{\partial^2 TC}{\partial T_1^2} \end{bmatrix}$$

If $H_{11} > 0$ and $H_{22} > 0$ then our model is convex.

5. Numerical Illustrations

This concept applies to perishable products including fresh foods, dairy, and some bakery items. This model is validated by providing two numerical examples for trade credit policy issues. The values for parameters are taken from previous study with some modification.

Example-1 The input parameters are as follows-

Parameters	Values
a	100
b	0.001
c	0.3
d	0.005
A	45
γ	0.2
θ	0.001
M	100 days
f	10
o	200 ₹/order
α	4
ξ	0.7
C_h	100 /unit/unit time
C_h''	30 /unit/unit time
C_l	40 /unit/unit time
C_d	80 /unit/unit time
S_c	200 /unit/unit time
r	0.5
I_e	0.3 /₹/days
I_c	0.4 /₹/days
z	40
p	500 ₹/unit
C_{de}	60
C_{he}	90

The optimal solution for case-1 $T_1 = 89.76\ days, T = 146.54\ days$ and $total\ cost = ₹\ 559324$

Example-2 The input parameters are as follows-

Parameters	Values
a	100
b	0.001
c	0.3
d	0.005
A	45

γ	0.2
θ	0.001
M	180 days
f	10
o	200 ₹/order
α	4
ξ	0.7
C_h	100 /unit/unit time
C_h''	30 /unit/unit time
C_l	40 /unit/unit time
C_d	80 /unit/unit time
S_c	200 /unit/unit time
r	0.5
I_e	0.3 /₹/days
I_c	0.4 /₹/days
z	40
p	500 ₹/unit
C_{de}	60
C_{he}	90

The optimal solution for case-2 $T_1 = 99.6\ days, T = 179.6\ days$ and $total\ cost = ₹\ 555328$

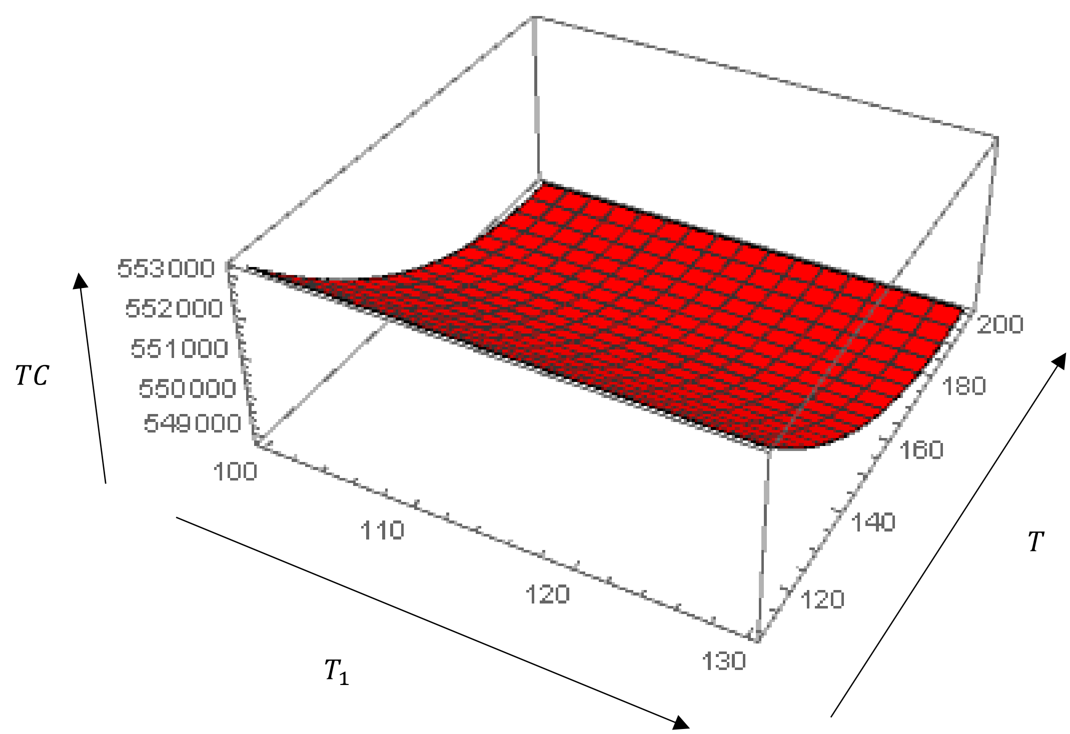

Fig-3 convexity between T and T_1 with respect to total cost

6. Sensitivity Analysis

To gain further insights into the behavior of the proposed inventory model, a sensitivity analysis is conducted by varying key parameters individually while keeping others constant. The optimal solution for case-2 is considered and effect of various parameters are shown in below table-

Table 2 Sensitivity Analysis of various parameters

Parameters	%change	T_1	T	Total cost
A	+20%	123.4	209.28	679823
	+10%	100.3	189.27	619723
	0	99.6	179.6	596328
	-10%	92.37	167.228	539872
	-20%	89.266	152.3	519722
γ	+20%	99.31	258.18	596328
	+10%	99.45	217.3	596328
	0	99.64	179.6	596328
	-10%	99.83	163.2	596328
	-20%	99.92	159.26	596328
θ	+20%	138.2	203.26	452972
	+10%	129.34	187.3	542861
	0	99.6	179.6	596328
	-10%	107.17	187.22	618623
	-20%	114.54	175.27	652972
M	+20%	91.76	209.37	668261
	+10%	95.76	197.3	638762
	0	99.6	179.6	596328
	-10%	99.80	172.36	568162
	-20%	99.97	167.26	518762
C_l	+20%	101.5	98.23	462567
	+10%	108.65	145.27	518232
	0	99.6	179.6	596328
	-10%	54.86	189.28	608272
	-20%	42.87	207.65	629817
I_c	+20%	79.65	283.7	596328
	+10%	87.75	267.37	596328
	0	99.6	179.6	596328
	-10%	108.76	165.29	596328

	-20%	127.87	156.37	596328
C_d	+20%	198.64	98.187	594267
	+10%	158.53	108.37	595276
	0	99.6	179.6	596328
	-10%	59.7	189.3	597757
	-20%	48.76	205.37	599816
I_e	+20%	76.98	181.2	782721
	+10%	85.76	180.2	692879
	0	99.6	179.6	596328
	-10%	100.76	178.236	507628
	-20%	137.65	176.27	489286
S_c	+20%	87.54	298.37	782682
	+10%	91.75	276.37	688123
	0	99.6	179.6	596328
	-10%	101.2	139.28	827687
	-20%	120.3	126.37	978634
r	+20%	103.21	98.27	626145
	+10%	106.3	101.287	876278
	0	99.6	179.6	596328
	-10%	87.5	218.2	876213
	-20%	78.2	243.27	576259
p	+20%	109.27	183.2	596328
	+10%	107.287	181.98	596328
	0	99.6	179.6	596328
	-10%	87.276	177.3	596328
	-20%	85.76	175.28	596328
C_{de}	+20%	109.27	179.6	672876
	+10%	114.276	179.6	762676
	0	99.6	179.6	596328
	-10%	109.28	179.6	376282
	-20%	105.27	179.6	783687
C_{he}	+20%	119.2	179.6	626145
	+10%	115.27	179.6	876278
	0	99.6	179.6	596328
	-10%	84.3	179.6	876213
	-20%	82.17	179.6	576259

7. Observation

From the above table the following observation are obtained-

i. When increase in parameter A then T_1 is increasing, total cycle length is also increasing and total cost again increasing.
ii. When increase in parameter γ then T_1 is slightly decreasing, total cycle length is increasing and total cost is constant.
iii. When increase in parameter θ then T_1 is fluctuating, total cycle length is increasing and total cost is decreasing.
iv. When increase in parameter M then T_1 is decreasing, total cycle length is increasing and total cost is also increasing.
v. When increase in parameter C_l then T_1 is increasing, total cycle length is decreasing and total cost is also decreasing.
vi. When increase in parameter I_c then T_1 is decreasing, total cycle length is increasing and total cost is constant.
vii. When increase in parameter C_d then T_1 is increasing, total cycle length is decreasing and total cost is also decreasing.
viii. When increase in parameter I_e then T_1 is decreasing, total cycle length is increasing and total cost is also increasing.
ix. When increase in parameter S_c then T_1 is decreasing, total cycle length is increasing and total cost is fluctuating.
x. When increase in parameter r then T_1 is increasing, total cycle length is decreasing and total cost is fluctuating.
xi. When increase in parameter p then T_1 is increasing, total cycle length is also increasing and total cost is constant.
xii. When increase in parameter C_{de} then T_1 is fluctuating, total cycle length is constant and total cost is also fluctuating.
xiii. When increase in parameter C_{he} then T_1 is increasing, total cycle length is constant and total cost is also increasing.

8. Conclusion

This study develops a Deterioration Control Decision Support System (DCDSS) for retailers managing highly perishable goods specifically branded organic fruits and vegetables under trade credit and partial shortage conditions. The proposed model incorporates a realistic and dynamic demand function that depends on time, selling price, advertisement effort, and product deterioration rate. It also accounts for the financial impact of a supplier's credit period and differentiates between interest earned on sold items and interest charged on unsold inventory. A key contribution of the model lies in its integration of controllable deterioration through preservation technology. By linking quality improvement efforts to marginal holding costs via a freshness quality indicator, the model enables retailers to strike an optimal balance between product freshness, cost minimization, and demand stimulation. Additionally, the inclusion of partial backlogging provides flexibility in managing shortages, reflecting actual consumer behaviour in perishable goods markets. Sensitivity analysis confirms that the most influential factors affecting the retailer’s total cost are the deterioration control cost, trade credit duration, advertisement intensity, and price sensitivity. These findings offer practical insights for decision-makers, suggesting that coordinated strategies in pricing, marketing, preservation, and financing are essential to optimize performance. Future research may extend this model by incorporating stochastic demand, multiple items, dynamic pricing, and real-time shelf-life tracking technologies. Nonetheless, the current model provides a valuable analytical foundation and practical framework for improving profitability and sustainability in perishable product retailing.

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